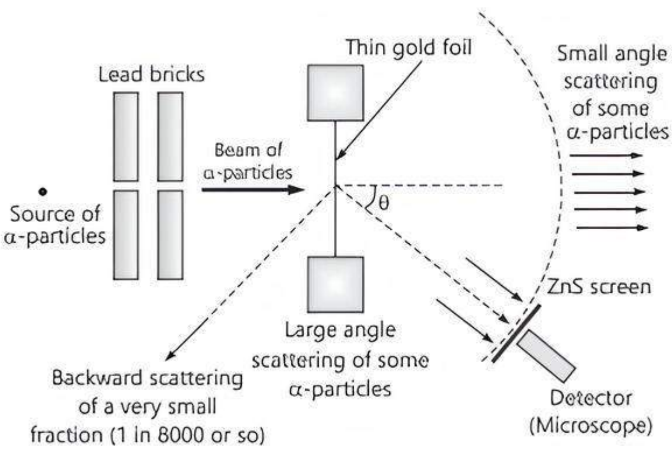
12 Atoms

Fastrack Revision

- **Atom:** The smallest invisible particle of an element that can exist is known as atom. Atom is electrically neutral, therefore it contains equal amount of positive and negative charges.
- **Thomson's Model of an Atom:** An atom consists of positively charged matter in which the negatively charged electrons are uniformly embedded like plums in a pudding. This model could not explain scattering of α-particles through thin foils and hence discarded.
- **Alpha-particle Scattering Experiment:** At the suggestion of Ernst Rutherford in 1911, H. Geiger and E. Marsden performed this experiment. The following figure shows a schematic diagram of this experiment.



Schematic arrangement of the Geiger-Marsden experiment.

In this experiment, alpha-particles emitted by a source were collimated into a narrow beam by their passage through lead bricks. The beam was allowed to full on a thin foil of gold thickness 2.1×10^{-7} m.

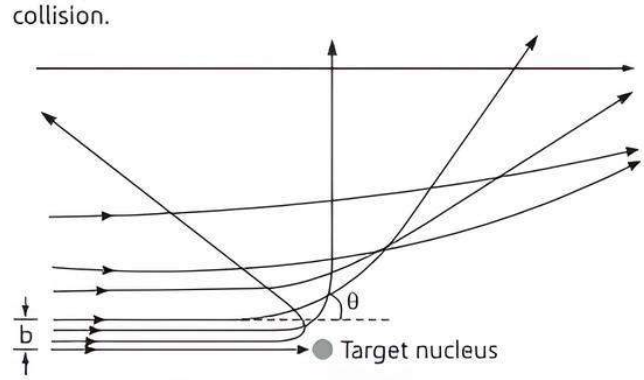
The scattered alpha-particles were observed through a rotatable detector consisting of zinc sulphide (ZnS) screen and a microscope. The scattered α-particles on striking the screen produced brief light flashes or scintillation viewed by a microscope.

- **Rutherford's Model of an Atom:** Geiger and Marsden in their experiment on scattering of alpha-particles found that most of the alpha-particles passed undeviated through thin foils but some of them were scattered through very large angles.
  From the results of these experiments, Rutherford proposed the following model of an atom:
  - An atom consists of a small and massive central core in which the entire positive charge and almost the whole mass of the atom are concentrated. This core is called the nucleus.
  - The nucleus occupies a very small space as compared to the size of the atom.
  - The atom is surrounded by a suitable number of electrons so that their total negative charge is equal to the total positive charge on the nucleus and the atom as a whole is electrically neutral.
  - The electrons revolve around the nucleus in various orbits just as planets revolve around the sun.
  - The centripetal force required for their revolution is provided by the electrostatic attraction between the electrons and the nucleus.
- **Drawbacks of Rutherford Model**
  - This model could not explain stability of the atom because according to classical electromagnetic theory the electron revolving around the nucleus must continuously radiate energy revolving around the nucleus in the form of electromagnetic radiation and hence it should fall into the nucleus.
  - This model cannot explain the characteristic line spectra of atoms of different elements.
- **Distance of Closest Approach:** When an alpha-particle of mass m and velocity v moves directly towards a nucleus of atomic number Z, its initial energy E, which is just the kinetic energy K, gets completely converted into potential energy U at stopping point. This stopping point happens to be at a distance of closest approach d from the nucleus.

$$E = K = \frac{1}{2}mv^2 = \frac{1}{4\pi\varepsilon_0}\frac{2eZe}{d} = \frac{2Ze^2}{4\pi\varepsilon_0 d}$$

Hence, $$d = \frac{2Ze^2}{4\pi\varepsilon_0 K}$$

- **Alpha-particle Trajectory:** The trajectory traced by an alpha-particle depends on the impact parameter (b) of collision.



Trajectory of α-particles in the coulomb field of a target nucleus.

  - **Impact Parameter:** It is the perpendicular distance of the initial velocity vector of the α-particle from the centre of the nucleus as shown in figure.
  - **Scattering Angle:** It is the angle (θ) by which α-particle gets deviated from its original path around the nucleus.
    Rutherford deduced the following relationship between the impact parameter b and the scattering angle θ:

$$b = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2 \cot\frac{\theta}{2}}{E}$$

- **Electron Orbits:** For a dynamically stable orbit in a hydrogen atom,

$$F_e = F_c$$

where F_e = electrostatic force

F_c = centripetal force

$$\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} = \frac{mv^2}{r}$$

$$r = \frac{e^2}{4\pi\varepsilon_0 mv^2}$$

The kinetic energy (k) and electrostatic potential energy (U) of the electron in hydrogen atom are

$$K = \frac{1}{2}mv^2 = \frac{e^2}{8\pi\varepsilon_0 r} \text{ and } U = \frac{-e^2}{4\pi\varepsilon_0 r}$$

Thus, the total energy E of the electron in a hydrogen atom is

$$E = K + U = \frac{e^2}{8\pi\varepsilon_0 r} - \frac{e^2}{4\pi\varepsilon_0 r}$$

$$E = \frac{-e^2}{8\pi\varepsilon_0 r}$$

The total energy of the electron is negative. This implies the fact that the electron is bound to the nucleus. If E were positive, an electron will not follow a closed orbit around the nucleus.

- **Quantisation or Discretisation:** The quantisation or discretisation of a physical quantity means that it can not vary continuously to have any arbitrary value but can change only discontinuously to take certain specific values.
- **Bohr's Model for the Hydrogen Atom:** Bohr combined classical and early quantum concepts and gave his theory in the form of three postulates.
  - **Stationary Orbits:** While revolving in the permissible orbits, an electron does not radiate energy. These non-radiating orbits are called stationary orbits.
  - **Quantum Condition:** Of all the possible circular orbits allowed by the classical theory, the electrons are permitted to circulate only in such orbits in which the angular momentum of an electron is an integral multiple of $h/2\pi$, where h being Planck's constant.

    $\therefore$ Angular momentum, $L = mvr = \dfrac{nh}{2\pi}$

    where, n is called principal quantum number and values $n = 1, 2, 3, \ldots\ldots$.
  - **Frequency Condition:** An atom can emit or absorb radiation in the form of discrete energy photons only, when an electron jumps from a higher to a lower orbit or from a lower to a higher orbit. If E_1 and E_2 are the energies associated with these permitted orbits then the frequency of the emitted absorbed radiation is,

    $$h\nu = E_2 - E_1$$
- **Some Terms Related to Bohr's Model of the Hydrogen Atom**
  - **Velocity of an Electron in the nth Orbit:**

    $$v = \frac{2\pi ke^2}{nh} = \alpha\frac{c}{n} = \frac{1}{137}\frac{c}{n}$$

    where, $\alpha = \dfrac{2\pi ke^2}{ch}$ is fine structure constant.
  - **Radius of nth Possible Orbit:**

    $$r = \left(\frac{n^2}{m}\right)\left(\frac{h}{2\pi}\right)^2\frac{4\pi\varepsilon_0}{e^2}$$
  - **Energy of an Electron in nth Orbit:**

    $$E_n = \frac{2\pi^2 mk^2Z^2e^4}{n^2h^2} = -\frac{13.6}{n^2}\text{eV}$$

    Here negative sign shows that electron is bound with the nucleus.
- **The Line Spectra of the Hydrogen Atom:** According to the third postulate of Bohr's model, when an atom makes a transition from the higher energy state with quantum number n_i to the lower energy state with quantum number $n_f (n_f < n_i)$. the difference of energy is carried away by a photon of frequency ν_{if} such that

$$h\nu_{if} = E_{n_i} - E_{n_f}$$

$$h\nu_{if} = \frac{me^4}{8\varepsilon_0^2h^2}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) \quad \ldots(1)$$

or

$$\nu_{if} = \frac{me^4}{8\varepsilon_0^2h^3}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

Eq. (1) is the Rydberg formula, for the spectrum of the hydrogen atom. In this relation, if we take $n_f = 2$ and $n_i = 3, 4, 5, \ldots$

The Rydberg constant R is readily identified to be

$$R = \frac{me^4}{8\varepsilon_0^2h^3c} \quad \ldots(2)$$

If we insert the values of various constants in Eq. (2), we get

$$R = 1.03 \times 10^7 \text{ m}^{-1}$$

This is a value very close to the value $(1.097 \times 10^7 \text{ m}^{-1})$ obtained from the empirical Balmer formula.

- **de-Broglie's Explanation of Bohr's Second Postulate of Quantisation:** de-Broglie hypothesis provided on explanation for Bohr's second postulate for the quantisation of angular momentum of orbiting electron. The quantised electron orbits and energy states are due to the wave nature of electron and only resonant standing waves can persist.
- **Failure of Bohr's Model:**
  - This model is applicable only to hydrogen-like atoms and fails in case of higher atoms.
  - It could not explain the fine structure of the spectral lines in the spectrum of hydrogen atom.
  - Bohr's model is unable to account for intensity variations.
  - This model does not explain why orbits of electrons are taken as circular whereas elliptical orbits are also possible.

Practice Exercise

Multiple Choice Questions

Q 1. When α-particles are sent through a thin gold foil, most of them go straight through the foil, because:
a. α-particles are positively charged
b. mass of α-particle is more than mass of electron
c. most of the part of an atom is empty space
d. α-particles moves with high velocity

Q 2. The existence of positively charged nucleus was established by:
a. Bohr's model of H-atom
b. positive ray analysis
c. α -scattering experiment
d. Thomson's model of atom

Q 3. In an experiment of scattering of α-particle showed for the first time that the atom has:
a. electron b. proton
c. neutron d. nucleus

Q 4. In Geiger-Marsden experiment, the expression of distance of closest approach to the nucleus of an alpha-particle before it comes to momentarily at rest and reverse its direction is:
a. $\frac{Ze^2}{4\pi\varepsilon_0 K}$ b. $\frac{Ze^2}{2\varepsilon_0 K}$
c. $\frac{Ze^2}{2\pi\varepsilon_0 K}$ d. $\frac{Ze^2}{4\varepsilon_0 K}$

Q 5. What was the order of thickness of gold foil on which beam of α-particles allowed to fall in Geiger-Marsden experiment?
a. 10^{-3} m b. 10^{-9} m
c. 10^{-7} m d. 10^{-5} m

Q 6. Which of the following statements is not correct according to Rutherford model?
a. Most of the space inside an atom is empty
b. The electrons revolve around the nucleus under the influence of coulomb force acting on them
c. Most part of the mass of the atom and its positive charge are concentrated at its centre
d. The stability of atom was established by the model

Q 7. When alpha particles are sent through a thin gold foil, most of them go straight through the foil, because: *(CBSE SQP 2023-24)*
a. alpha particles are positively charged
b. the mass of an alpha particle is more than the mass of an electron
c. most of the part of an atom is empty space
d. alpha particles move with high velocity

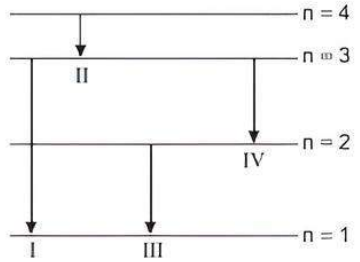
Q 8. The radius of an atomic nucleus have an order of:
a. 10^{-8}m b. 10^{-15}m
c. 10^{-12}m d. 10^{-10}m

Q 9. According to Bohr's postulates, an electron revolve around the nucleus in .... orbits.
a. dynamic b. stationary
c. lower d. first

Q 10. The energy of an electron in nth orbit of hydrogen atom is $E_n = -13.6/n^2$ eV. The negative sign of energy indicates that: *(CBSE SQP 2023-24)*
a. electron is free to move.
b. electron is bound to the nucleus.
c. kinetic energy of electron is equal to potential energy of electron.
d. atom is radiating energy

Q 11. An electron with angular momentum L moving around the nucleus has a magnetic moment given by: *(CBSE SQP 2023-24)*
a. $e\,L/2m$ b. $e\,L/3m$
c. $e\,L/4m$ d. $e\,L/m$

Q 12. The diagram shows four energy level of an electron in Bohr model of hydrogen atom. Identify the transition in which the emitted photon will have the highest energy. *(CBSE 2023)*



a. I b. II c. III d. IV

Q 13. The radius of the nth orbit in Bohr model of hydrogen atom is proportional to: *(CBSE 2023)*
a. n^2 b. $\frac{1}{n^2}$
c. n d. $\frac{1}{n}$

Q 14. The relationship between kinetic energy (K) and potential energy (U) of electron moving in an orbit around the nucleus is:
a. $U = -K$ b. $U = -2K$
c. $U = -3K$ d. $U = -\frac{1}{2}K$

Q 15. The value of ionisation energy of the hydrogen atom is:
a. 12.09 eV b. 10.6 eV
c. 13.6 eV d. 4.34 eV

Q 16. The energy required to excite an electron in hydrogen atom to its first excited state is:
a. 8.5 eV b. 10.2 eV
c. 12.7 eV d. 13.6 eV

Q 17. What is the order of velocity of electron in a hydrogen atom in ground state?

a. $10^6 ms^{-1}$ b. $10^2 ms^{-1}$
c. $10^{10} ms^{-1}$ d. $10^9 ms^{-1}$

Q.18. The radius of the innermost electron orbit of a hydrogen atom is 5.3×10^{-11} m. The radius of the $n = 3$ orbit is: *(CBSE SQP 2022-23)*

a. 1.01×10^{-10} m b. 1.59×10^{-10} m
c. 2.12×10^{-10} m d. 4.77×10^{-10} m

Q 19. The number of de-Broglie wavelengths contained in the second Bohr orbit of hydrogen atom is:

a. 4 b. 3
c. 2 d. 1

Assertion & Reason Type Questions

Directions (Q.Nos. 20-26): *In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as:*

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).
c. Assertion (A) is true but Reason (R) is false.
d. Both Assertion (A) and Reason (R) are false.

Q 20. Assertion (A): The force of repulsion between atomic nucleus and alpha-particle varies with distance according to inverse square law.

Reason (R): Rutherford did alpha-particle scattering experiment.

Q 21. Assertion (A): The positively charged nucleus of an atom has a radius of atmost 10^{-15} m.

Reason (R): In alpha-particle scattering experiment, the distance of closest approach for alpha-particles $\simeq 10^{-15}$ m.

Q 22. Assertion (A): For the scattering of alpha-particles at a large angle, only the nucleus of the atom is responsible.

Reason (R): Nucleus is very heavy in comparison to electrons.

Q 23. Assertion (A): Bohr had to postulate that the electrons in stationary orbits around the nucleus do not radiate.

Reason (R): According to classical Physics, all moving electrons radiate.

Q 24. Assertion (A): Electrons in the atom are held due to coulomb forces.

Reason (R): The atom is stable only because the centripetal force due to Coulomb's law is balanced by the centrifugal force.

Q 25. Assertion (A): The whole mass of the atom is considered in the nucleus.

Reason (R): The mass of a nucleus can be either less than or more than the sum of the masses of nucleons present in it.

Q 26. Assertion (A): It is essential that all the lines available in the emission spectrum will also be available in the absorption spectrum.

Reason (R): The spectrum of hydrogen atom is only absorption spectrum.

Fill in the Blanks Type Questions

Q 27. The angle of scattering θ for zero value of impact parameter b is ....

Q 28. An α-particle contains .... protons and .... neutrons.

Q 29. The force responsible for scattering of alpha-particle with target nucleus is ....

Q 30. The scattering angle will decreases with the .... in impact parameter.

Q 31. The SI unit of impact parameter is ....

Q 32. The Rutherford's model of an atom cannot explain the characteristics .... spectrum emitted by H-atom.

Q 33. The frequency spectrum of radiation emitted as per Rutherford's model of atom is ....

Q 34. According to the Rutherford's model of an atom, the most of space in atom is ....

Q 35. According to Bohr's atomic model, the circumference of the electron orbit is always an .... multiple of de-Broglie wavelength.

Q 36. If the size of first orbit of hydrogen atom is 0.5 Å, the size of 2nd orbit of hydrogen atom would be ....

Answers

1. (c) most of the part of an atom is empty space
2. (c) α-scattering experiment
3. (d) nucleus
4. (c) $\dfrac{Ze^2}{2\pi E_0 K}$
5. (c) 10^{-7}m
6. (d) The stability of atom was established by the model.
7. (c) most of the part of an atom is empty space

In Rutherford's alpha scattering experiment. when we pass the alpha particle on gold foil there are a few observations which is recorded as:

(i) Most of the particles remain undeflected
(ii) Some of the particles are deflected through a small angles.
(iii) Very few particles are deflected by 180° and came back

So. we can conclude from that atom has most space as empty.

8. (b) 10^{-15}m
9. (b) stationary
10. (b) electron is bound to the nucleus
11. (a) $eL/2m$
12. (c) III

Clearly, we can see, the transition between the energy levels $n = 2$ to $n = 1$ will be maximum. Thus, the transition represented by III will emit the highest energy photon.

13. (a) n^2.
14. (b) $U = -2K$

Kinetic energy, $K = \frac{1}{2}mv^2 = \frac{e^2}{8\pi\varepsilon_0 r}$

and potential energy, $U = -\frac{e^2}{4\pi\varepsilon_0 r}$

$\Rightarrow \quad U = -2K$

15. (c) 13.6 eV

The minimum energy required to free the electron from the ground state of the hydrogen atom is called the ionisation energy of hydrogen atom. Its value is 13.6 eV.

16. (b) 10.2 eV

As we know that,

$$E_n = \frac{-13.6}{n^2}\text{eV}$$

In ground state, energy

$$E_1 = \frac{-13.6}{1^2} = -13.6\text{ eV}$$

and in first excited state,

$$E_2 = \frac{-13.6}{2^2} = -3.4\text{ eV}$$

Hence, required energy $= E_2 - E_1$

$= -3.4 - (-13.6) = 10.2\text{ eV}$

17. (a) 10^6ms^{-1}
18. (d) 4.77×10^{-10} m

Given; radius of innermost orbit, $r_1 = 5.3 \times 10^{-11}$ m

Let r_3 be the radius of $n = 3$ orbit.

Then, $r_3 = n^2 r_1 = (3)^2 \times 5.3 \times 10^{-11}$ m

$= 4.77 \times 10^{-10}$ m

19. (c) 2

As we know, $2\pi r_n = n\lambda$

Given, $n = 2$

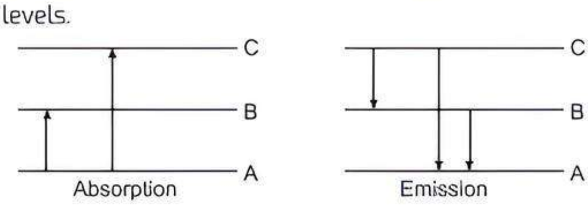
$\therefore \quad 2\pi r_2 = 2\lambda$

$= 2 \times$ de-Broglie wavelength

20. (b) Rutherford confirmed the repulsive force on α-particle due to nucleus varies with distance according to inverse square law and that the positive charges are concentrated at the centre and not distributed throughout the atom.
21. (a) Experimentally, it is found that the average radius of a nucleus is given by $R = R_0 A^{1/3}$, where $R_0 = 1.1 \times 10^{-15}\text{m} = 1.1\text{ fm}$.
22. (a) We know that, an electron is very light particle as compared to an α-particle. Hence, electron cannot scatter the α-particle at large angles, according to law of conservation of momentum. On the other hand, mass of nucleus is comparable with the mass of α-particle, hence only the nucleus of atom is responsible for scattering of α-particles.
23. (b) Bohr postulated that electrons in stationary orbits around the nucleus do not radiate.

This is one of the Bohr's postulate. According to this, the moving electrons radiate only when they go from one orbit to the next lower orbit.

24. (c) According to postulates of Bohr's atomic model, the electron revolve round the nucleus in fixed orbit of definite radii. As long as the electron is in a certain orbit, it does not radiate any energy.
25. (c) The whole mass of the atom is concentrated at nucleus and $M_{nucleus}$ < (Sum of the masses of nucleons) because, when nucleons combines, some energy is wasted.
26. (d) Emission transition can take place between any higher energy level and any energy level below it while absorption transitions start from the lowest energy level only and may end at any higher energy level. Hence, number of absorptions transition between two given energy levels is always less than the number of emission transition between same two levels.

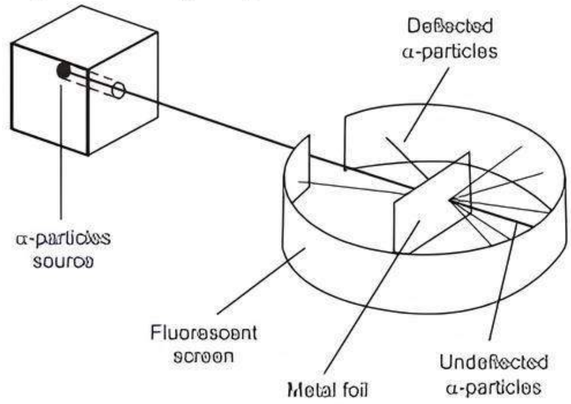


27. 180°
28. two, two
29. electrostatic force
30. increase
31. metre
32. line
33. continuous
34. empty
35. integral
36. 2Å

Case Study Based Questions

Case Study 1

A radioactive source emitting alpha-particles was enclosed within a protective lead shield. The radiation was focused into a narrow beam after passing through a slit in a lead screen. A thin section of gold foil was placed in front of the slit and a screen coated with zinc sulphide to render it fluorescent served as a counter to detect alpha-particles. As each alpha-particle struck the fluorescent screen, it produced a burst of light called a scintillation, which was visible through a viewing microscope attached to the back of the screen. The screen itself was movable, allowing to determine whether or not any α-particles were being deflected by the gold foil.



Read the given passage carefully and give the answer of the following questions:

Q 1. The particles which were deflected backwards in Rutherford's experiment were hit upon by:

a. nucleus
b. empty space
c. electrons
d. protons

Q 2. According to the Rutherford's atomic model, the whole atom is:

a. positively charged
b. negatively charged
c. neutral
d. None of these

Q 3. Rutherford in his atomic model could not explain the behaviour of which of the following?

a. Proton
b. Neutron
c. Electron
d. Neutrino

Q 4. Electron revolves around the nucleus in orbits which have:

a. variable energy
b. fixed energy
c. infinite energy
d. zero energy

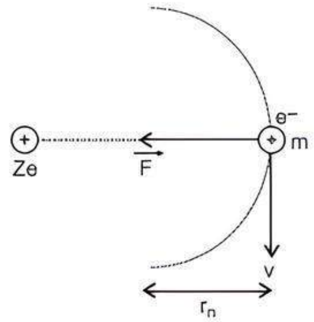
Answers

1. a. nucleus
2. c. neutral
3. c. Electron
4. b. fixed energy

Case Study 2

Hydrogen is the simplest atom of nature. There is one proton in its nucleus and an electron moves around the nucleus in a circular orbit. According to Niels Bohr, this electron moves in a stationary orbit. When this electron is in the stationary orbit, it emits no electromagnetic radiation. The angular momentum of the electron is quantized, *i.e.*, $mvr = (nh/2\pi)$, where m = mass of the electron, v = velocity of the electron in the orbit, r = radius of the orbit and $n = 1, 2, 3, ...$. When transition takes place from kth orbit to jth orbit, energy photon is emitted. If the wavelength of the emitted photon is λ, we find that $\frac{1}{\lambda} = R\left[\frac{1}{j^2} - \frac{1}{k^2}\right]$, where R is Rydberg's constant.

On a different planet, the hydrogen atom's structure was somewhat different from ours. The angular momentum of electron was $P = 2n(h/2\pi)$, *i.e.*, an even multiple of $(h/2\pi)$.



Read the given passage carefully and give the answer of the following questions:

Q 1. What is the minimum permissible radius of the orbit?

Q 2. In our world, the velocity of electron is v_0, when the hydrogen atom is in the ground state. Find the velocity of electron in this state on the other planet.

Q 3. In our world, the ionisation potential energy of a hydrogen atom is 13.6 eV. What will be the ionisation potential energy on the other planet?

Q 4. What is the total energy (E_n) of the electron in the stationary states in the nth orbit of the hydrogen atom?

Answers

1. On other planet, $mvr = 2n\frac{h}{2\pi} \Rightarrow v = \frac{nh}{\pi mr}$

$$\Rightarrow \frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} \Rightarrow \frac{mn^2h^2}{\pi^2 m^2 r^3} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}$$

Putting n = 1, we get $r = \frac{4h^2\varepsilon_0}{m\pi e^2}$

2. On our planet, $v_0 = \frac{e^2}{2\varepsilon_0 nh}$

On other planet, $v = \frac{e^2}{2\varepsilon_0(2n)h} = \frac{v_0}{2}$

3. On our planet, $E_n = -\frac{13.6}{n^2}$

On other planet, $E_n' = -\frac{13.6}{(2n)^2}$

$$\Rightarrow E_n' = \frac{E_n}{4} = \frac{13.6}{4} = 3.4 \text{ eV}$$

4. $\frac{-13.6}{n^2}$ eV

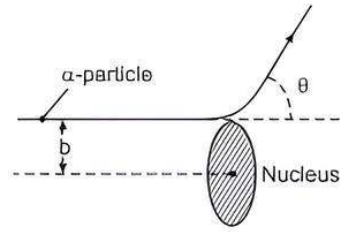
Very Short Answer Type Questions

Q 1. Suppose you are given a chance to repeat the α-particle scattering experiment using a thin sheet of solid hydrogen in place of the gold-foil. (Hydrogen is solid at temperature below 14 K.) What result do you expect? *(NCERT EXERCISE)*

Ans. Hydrogen nuclei (proton) are very light from α-particles. Hence α-particles will not be scattered by solid hydrogen. They will pass the hydrogen sheet without deviation.

Q 2. Show the trajectory of the alpha-particle when it approaches an atom of atomic number Z. *(CBSE 2017)*

Ans.



COMMON ERR(!)R

Students are often unable to recall the diagram.

Q 3. What is impact parameter?

Ans. Impact parameter is the perpendicular distance of the initial velocity vector of the α-particle from the centre of the nucleus, when it is far away from the atom.

Impact parameter, $b = \dfrac{1}{4\pi\varepsilon_0} \dfrac{Ze^2 \cot\frac{\theta}{2}}{E}$

Q 4. Name the experiment responsible for the discovery of atomic nucleus.

Ans. Rutherford's α-scattering experiment.

Q 5. The KE of α-particle incident on gold foil is doubled. How does the distance of closest approach change? *(CBSE 2017, 15)*

Ans. Distance of closest approach,

$$d = \frac{1}{4\pi\varepsilon_0}\left(\frac{2Ze^2}{E_K}\right) \Rightarrow d \propto \frac{1}{E_K}$$

Hence, distance of closest approach will be halved when KE is doubled.

Q 6. In the Rutherford's scattering experiment, the distance of closest approach for an α-particle is d_0. If α-particle is replaced by a proton, how much kinetic energy in comparison to α-particle will it require to have the same distance of closest approach d_0?

Sol. Energy of α-particle, $E_{K_\alpha} = \dfrac{1}{4\pi\varepsilon_0} \dfrac{(Ze)(2e)}{d_0}$

and energy of proton, $E_{K_p} = \dfrac{1}{4\pi\varepsilon_0} \dfrac{(Ze)(e)}{d_0}$

$$\Rightarrow \quad E_{K_p} = \frac{1}{2} E_{K_\alpha}$$

Q 7. Determine the distance of closest approach when an alpha-particle of kinetic energy 4.5 MeV strikes a nucleus of $Z = 80$, stops and reverses its direction. *(CBSE 2015)*

Sol. Distance of closest approach, $r_0 = \dfrac{1}{4\pi\varepsilon_0} \dfrac{(2Ze^2)}{E_K}$

$$= 9\times10^9 \times \frac{2\times80\times(1.6\times10^{-19})^2}{4.5\times10^6\times1.6\times10^{-19}} = 5.12\times10^{-14}\text{ m}$$

TiP

Students should practice enough numericals related to closest approach for better understanding.

Q 8. A proton and an electron have equal speeds. Find the ratio of de-Broglie wavelengths associated with them. *(CBSE 2020)*

Sol. We know that, $\lambda = \dfrac{h}{mv}$

So, $\dfrac{\lambda_p}{\lambda_e} = \dfrac{h/m_p v}{h/m_e v} = \dfrac{m_e}{m_p}$

But $m_p = 1.67\times10^{-27}$ kg

and $m_e = 9.1\times10^{-31}$ kg

So, $\dfrac{\lambda_p}{\lambda_e} = \dfrac{9.1\times10^{-31}}{1.67\times10^{-27}}$

$\Rightarrow \quad \lambda_p : \lambda_e = 5.44\times10^{-4} : 1 = 1 : 1838$

Q 9. The ground state energy of hydrogen atom is –13.6 eV. What are the kinetic and potential energies of electron in this state?

Sol. Given, total ground state energy = –13.6 eV

$\therefore$ Kinetic energy = – Total ground state energy

= –(–13.6 eV) = 13.6 eV

Potential energy = 2 × Total energy

= 2 × (–13.6) = –27.2 eV

Q 10. What is the maximum number of spectral lines emitted by a hydrogen atom when it is in the third excited state?

Sol. Number of spectral lines obtained due to transition of electron from $n = 4$ (3rd excited state) to $n = 1$ (ground state) is according to formula,

$$N = \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = \frac{4\times3}{2} = 6$$

Q 11. Define ionisation energy. What is its value for a hydrogen atom?

Ans. It is defined as the energy required to remove an electron from an atom, *i.e.*, the energy required to take an electron from its ground state to the outermost orbit ($n = \infty$).

Ionisation energy for hydrogen atom = $E_\infty - E_i$

= – (–13.6 eV) = + 13.6 eV

Q 12. A difference of 2.3 eV separates two energy-levels in an atom. What is the frequency of radiation emitted when the atom makes transition from the upper level to the lower level? *(NCERT EXERCISE)*

Sol. According to Bohr's postulate, $E_2 - E_1 = h\nu$

$\therefore$ Frequency of emitted radiation, $\nu = \frac{E_2 - E_1}{h}$

$$= \frac{2.3\text{ eV}}{h} = \frac{2.3 \times 1.6 \times 10^{-19}\text{ J}}{6.63 \times 10^{-34}\text{ Js}}$$

$$= 5.55 \times 10^{14}\text{ Hz.}$$

Short Answer Type-I Questions

Q 1. Define the distance of closest approach. An α-particle of kinetic energy '*K*' is bombarded on a thin gold foil. The distance of the closest approach is '*r*'. What will be the distance of closest approach for a α-particle of double the kinetic energy? *(CBSE 2017)*

Ans. At a certain distance *r* from the nucleus, whole of the KE of α-particle goes on converting into electrostatic potential energy and α-particle cannot go farther close to nucleus, this distance (r) is called distance of closest approach.

$$r = \frac{1}{4\pi\varepsilon_0} \cdot \frac{4Ze^2}{mv^2} \quad ...(1)$$

Kinetic energy of α -particle is given as

$$K = \frac{1}{4\pi\varepsilon_0} \frac{2e \cdot Ze}{r^2} \quad ...(2)$$

From eqs. (1) and (2), we get

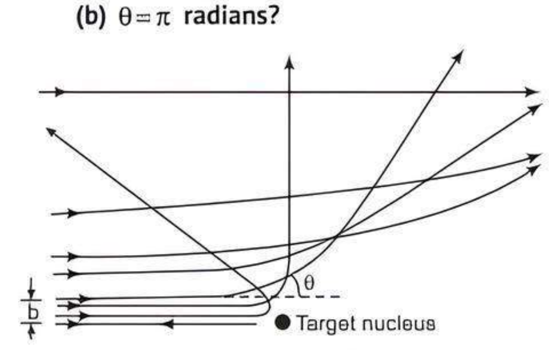
$$r \propto \frac{1}{K}$$

It is clear from the above expression, when K is doubled, r becomes half.

Q 2. The trajectories, traced by different α-particles, in Geiger-Marsden experiment were observed as shown in figure.

(i) What names are given to the symbols '*b*' and 'θ' shown here?

(ii) What can we say about values of *b* for (a) $\theta = 0°$ (b) $\theta = \pi$ radians?



Ans. (i) Symbol '*b*' represents impact parameter and 'θ' represents scattering angle.

(ii) Impact parameter, $b = \frac{Ze^2 \cot\theta/2}{4\pi\varepsilon_0\left(\frac{1}{2}mv^2\right)}$

(a) When $\theta = 0°$, b is maximum and represent the atomic size.

(b) When $\theta = \pi$ radians, b is minimum and represent nuclear size.

Q 3. Write two important limitations of Rutherford nuclear model of the atom. *(CBSE 2017)*

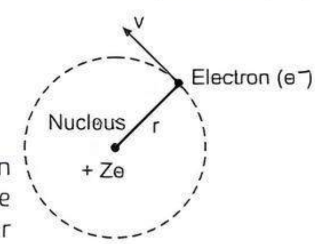
Ans. Limitations of Rutherford Nuclear Model:

(i) Inconsistent: As the revolving electrons looses energy continuously, it must spiral inwards and eventually fall into the nucleus.

(ii) Spectrum: Atoms should emit continuous spectrum but what we observe is only a line spectrum.

Q 4. Using Bohr's atomic model, derive an expression for the radius of *n*th orbit of the revolving electron in a hydrogen atom. *(CBSE 2023, 20)*

Ans. Let e, m and v be respectively the charge, mass and velocity of the electron and r be the radius of the orbit.



The positive charge on the nucleus is Ze, where Z is the atomic number (In case of hydrogen atom, $Z = 1$). As the centripetal force is provided by the electrostatic force of attraction, we have

$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{(Ze) \times e}{r^2}$$

or $$mv^2 = \frac{Ze^2}{4\pi\varepsilon_0 r} \quad ...(1)$$

From the first postulate of Bohr's atomic model, the angular momentum of the electron is

$$mvr = n\frac{h}{2\pi} \quad ...(2)$$

where, $n = 1, 2, 3, ...$ is principal quantum number.

From eqs. (1) and (2), we get $r = n^2 \frac{h^2 \varepsilon_0}{\pi m Z e^2}$

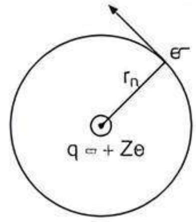
This is the equation for the radii of the permitted orbits.

COMMON ERROR

Derivation for the velocity of electron also have the common starting.

Q 5. Using Bohr's atomic model, derive the expression for the velocity of electron revolving in the *n*th orbit of hydrogen atom. *(CBSE 2020)*

Ans. Centripetal force = electrostatic attraction between nucleus and electron (e^-)



$$\Rightarrow \frac{mv_n^2}{r_n} = \frac{1}{4\pi\varepsilon_0} \frac{Ze \times e}{r_n^2}$$

$$\Rightarrow \quad mv_n^2 r_n = \frac{Ze^2}{4\pi\varepsilon_0} \quad ...(1)$$

By Bohr's second postulate, angular momentum of e^-,

$$mv_n r_n = \frac{nh}{2\pi} \quad ...(2)$$

Dividing eq. (1) by eq. (2), we get

$$\frac{mv_n^2 r_n}{mv_n r_n} = \frac{Ze^2}{4\pi\varepsilon_0} \times \frac{2\pi}{nh}$$

Hence, speed of electron, $v_n = \frac{Ze^2}{2\varepsilon_0 nh}$.

Q 6. What is the nuclear radius of ^{125}Fe, if that of ^{27}Al is 3.6 fermi? *(CBSE SQP 2022-23)*

Sol. From the relation $R = R_0 A^{1/3}$, where R_0 is a constant and A is the mass number of nucleus

$$R_{Fe}/R_{Al} = (A_{Fe}/A_{Al})^{1/3}$$
$$= (125/27)^{1/3}$$
$$R_{Fe} = 5/3\, R_{Al}$$
$$= 5/3 \times 3.6$$
$$= 6 \text{ fermi}$$

So, the nuclear radius of ^{125}Fe is 6 fermi.

Q 7. Calculate the orbital period of the electron in the first excited state of hydrogen atom. *(CBSE 2019)*

Sol. The velocity of electron, $v_n = \frac{1}{n}\frac{Ze^2}{2h\varepsilon_0}$

Here, $Z = 1$, $e = 1.6 \times 10^{-19}$ C, $\varepsilon_0 = 8.85 \times 10^{-12}$ NC^{2}m^{-2}

$h = 6.62 \times 10^{-34}$ J-s and $n = 2$ (in first excited state)

Putting the values, we get

$$v_2 = \frac{1 \times (1.6 \times 10^{-19})^2}{2 \times 2 \times (6.62 \times 10^{-34}) \times (8.85 \times 10^{-12})}$$
$$= 1.09 \times 10^6 \text{ m/s}$$

Radius of orbit, $r_2 = \frac{n^2 h^2 \varepsilon_0}{\pi m e^2}$

Here, $m = 9.1 \times 10^{-31}$ kg

Hence, $$r_2 = \frac{(2)^2 \times (6.62 \times 10^{-34})^2 (8.85 \times 10^{-12})}{3.14 \times (9.1 \times 10^{-31}) \times (1.6 \times 10^{-19})^2}$$
$$= 2.12 \times 10^{-10} \text{ m}$$

Time period or orbital period,

$$T = \frac{2\pi r_2}{v_2} = \frac{2 \times 3.14 \times 2.12 \times 10^{-10}}{1.09 \times 10^6} = 1.22 \times 10^{-15} \text{ s.}$$

TiP

Write all the values in starting to avoid any calculation error.

Q 8. Obtain the expression for the ratio of the de-Broglie wavelengths associated with the electron orbiting in the second and third excited states of hydrogen atom. *(CBSE 2019)*

Ans. As we know that, $r_n = r_0 n^2$

where, r_0 = Bohr's radius = 0.529 Å.

For second excited state $(n = 3)$, $r_3 = r_0 \times (3)^2$

For third excited state $(n = 4)$, $r_4 = r_0 \times (4)^2$

Again, $2\pi r_n = n\lambda_{(n-1)}$

Accordingly, $2\pi r_3 = 3\lambda_{(3-1)} = 3\lambda_2$

$$\Rightarrow \quad 2\pi[r_0(3)^2] = 3\lambda_2 \quad ...(1)$$

Similarly, $$2\pi[r_0(4)^2] = 4\lambda_3 \quad ...(2)$$

Dividing eq. (1) by eq. (2), we have

$$\frac{2\pi r_0(3)^2}{2\pi r_0(4)^2} = \frac{3\lambda_2}{4\lambda_3} \Rightarrow \frac{\lambda_2}{\lambda_3} = \frac{4}{3} \times \frac{3^2}{4^2} = \frac{3}{4}$$

Hence, $\lambda_2 : \lambda_3 = 3 : 4$

COMMON ERROR

Students often take $n = 3$ for second excited state and so on.

Q 9. State Bohr postulate of hydrogen atom that gives the relationship for the frequency of emitted photon in a transition. *(CBSE 2016)*

Or

State Bohr's postulate of hydrogen atom which successfully explains emission lines in the spectrum of hydrogen atom. *(CBSE 2015)*

Ans. Bohr's Postulate of Transition: When an electron makes a transition from higher energy level (E_2) to lower energy level (E_1), a photon is emitted which have the energy equal to the energy difference of two levels.

i.e., $h\nu = E_2 - E_1$

This equation is called Bohr's frequency condition.

Q 10. The radius of innermost orbit of a hydrogen atom is 5.3×10^{-11} m. What are the radii of $n = 2$ and $n = 3$ orbits? *(NCERT EXERCISE)*

Sol. The radii of Bohr's orbits are given by

$$r_n = \frac{\varepsilon_0 h^2 n^2}{\pi m e^2} \quad \text{or} \quad r_n \propto n^2$$

For ground state $n = 1$, $r_1 = 5.3 \times 10^{-11}$ m (given)

$$\frac{r_2}{r_1} = \left(\frac{n_2}{n_1}\right)^2 \quad \text{or} \quad r_2 = \left(\frac{2}{1}\right)^2 r_1$$
$$= 4r_1 = 4 \times 5.3 \times 10^{-11} = 2.12 \times 10^{-10} \text{ m}$$

For $n = 3$,

$$r_3 = (3)^2 r_1 = 9 \times 5.3 \times 10^{-11} = 4.77 \times 10^{-10} \text{ m}$$

Q 11. Consider two different hydrogen atoms. The electron in each atom is in an excited state. Is it possible for the electrons to have different energies but same orbital angular momentum according to the Bohr model? Justify your answer.

(CBSE SQP 2022 Term-2)

Ans. No; because according to Bohr's model, energy of electron in nth orbit of H-atom,

$$E_n = -\frac{13.6}{n^2}$$

Hence, electrons having different energies belong to different energy levels, *i.e.*, different values of n.

Therefore, their angular momentum will be different due to different values of n.

Angular momentum, $L = mvr = \frac{nh}{2\pi}$

Q 12. Find out the wavelength of the electron orbiting in the ground state of hydrogen atom. *(CBSE 2017)*

Sol. For an electron revolving in nth orbit of radius r_n then, we have

$$n\lambda = 2\pi rn$$

where, λ is the wavelength of electron.

For electron orbiting in ground state $n = 1$,

$$1\times\lambda = 2\pi r_n = 2\pi \times n^2 r_0 = 2\pi \times 0.5 Å = \pi Å$$

$$\lambda = 3.14 Å$$

TiP

For ground state $n = 1$

Q 13. An α-particle and a proton are accelerated through the same potential difference. Find the ratio of their de-Broglie wavelengths. *(CBSE 2017)*

Sol. From de-Broglie equation, we know that

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2mqV}}$$

$$\therefore \frac{\lambda_\alpha}{\lambda_p} = \left\{\frac{h}{\sqrt{2m_\alpha q_\alpha V}}\right\} \times \left\{\frac{\sqrt{2m_p q_p V}}{h}\right\}$$

$$\frac{\lambda_\alpha}{\lambda_p} = \frac{\sqrt{m_p q_p}}{\sqrt{m_\alpha q_\alpha}}$$

$$= \frac{\sqrt{m_p q_p}}{\sqrt{4m_p 2q_p}} = \frac{1}{2\sqrt{2}} \quad (\because m_\alpha = 4m_p, q_\alpha = 2q_p)$$

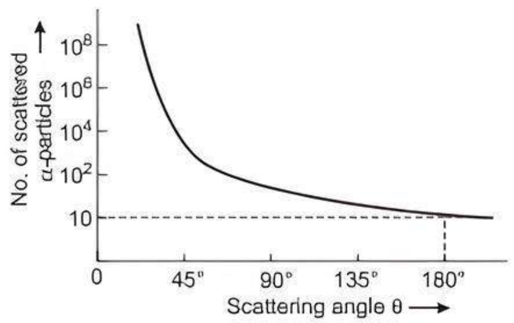
Hence, $\lambda_\alpha : \lambda_p = 1 : 2\sqrt{2}$

Short Answer Type-II Questions

Q 1. (i) Draw a graph to show the variation of the number of scattered particles detected (N) in Geiger-Marsden experiment as a function of scattering angle (θ). *(CBSE 2023)*

(ii) Discuss briefly two conclusions that can be drawn from this graph and how they lead to the discovery of nucleus in an atom?

Ans. (i) Graph showing the variation of the number of scattered particles:



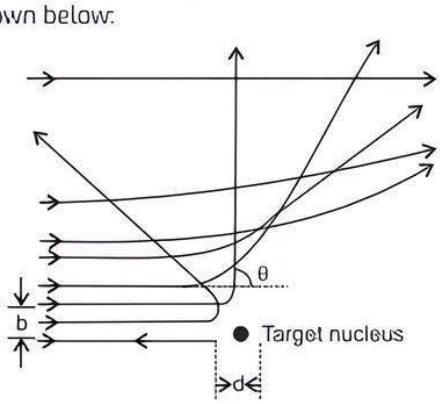
(ii) **Conclusions :** (a) Entire positive charge and most of the mass of the atom is concentrated in the nucleus with the electrons some distance away.

(b) Size of the nucleus is about 10^{-15}m to 10^{-14}m, while size of the atom is 10^{-10}m, so the electrons are at distance 10^4m to 10^5m from the nucleus and being large empty space in the atom, most particles go through the empty space.

Q 2. (i) In Rutherford scattering experiment, draw the trajectory traced by α-particles in the coulomb field of target nucleus and explain, how this led to estimate the size of the nucleus.

(ii) State Bohr's quantisation condition for defining stationary orbits.

Ans. (i) The trajectory, traced by the α-particles in the coulomb field of target nucleus, has the form as shown below:



The size of the nucleus was estimated by observing the distance (d) of closest approach, of the α-particles. This distance is given by:

$$d = \frac{1}{4\pi\varepsilon_0} \cdot \frac{(2e)(Ze)}{K}$$

where, K = kinetic energy of the α-particles when they are far away from the target nuclei.

(ii) **Quantum Condition:** The stationary orbits are those in which momentum of electron is an integral multiple of $h/2\pi$.

i.e., $$mvr = \frac{nh}{2\pi}$$

where $n = 1, 2, 3, \ldots$

Integer n is called the principal quantum number and this relation is called Bohr's quantum condition.

Q 3. (i) Explain briefly, how Rutherford scattering of α-particle by a target nucleus can provide information on the size of the nucleus?

(ii) Show that density of nucleus is independent of its mass number A. *(CBSE 2019)*

Ans. (i) According to Rutherford's experiment, following observations were made:

(a) Most of the α-particles passed through the gold foil without any appreciable deflection.

(b) Only 0.14% of incident α-particles scattered by more than 1°. But about 1 α-particle in every 8000 particles deflected by more than 90°.

Thus, all these leads to the conclusion that atom has a lot of empty space and practically the entire mass of the atom is confined to an extremely small centered core called nucleus, whose size is of the order from 10^{-15} m to 10^{-14} m.

(ii) Radius of nucleus, $r = r_0 A^{1/3}$

Also, we know that

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}} = \frac{mA}{\frac{4}{3}\pi r^3}$$

where, m = mass on a nucleus,

A = mass number of nucleus.

Again,

$$\text{Density} = \frac{mA}{\frac{4}{3}\pi (r_0 A^{1/3})^3}$$

$$= \frac{mA}{\frac{4}{3}\pi r_0^3 A} = \frac{3m}{4\pi r_0^3}$$

Hence, nuclear density is independent of A.

Q 4. Using Bohr's postulates, derive the expression for the total energy of the electron in the stationary states of the hydrogen atom.

Ans. Since, $mvr = \frac{nh}{2\pi}$ ....(1)

But centripetal force = electrostatic force

$$\therefore \quad \frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}$$

$$\Rightarrow \quad r = \frac{e^2}{4\pi\varepsilon_0 mv^2} \quad \text{....(2)}$$

From eqs. (1) and (2), we get

$$r = \frac{e^2}{4\pi\varepsilon_0 m\left(\frac{nh}{2\pi mr}\right)^2}$$

$$\Rightarrow \quad r = \frac{\varepsilon_0 n^2 h^2}{\pi m e^2}$$

Potential energy, $PE = -\frac{1}{4\pi\varepsilon_0}\cdot\frac{e^2}{r}$

$$= -\frac{me^4}{4\varepsilon_0^2 n^2 h^2}$$

Kinetic energy, $KE = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{nh}{2\pi mr}\right)^2$

$$= \frac{n^2h^2\pi^2m^2e^4}{8\pi^2 m\varepsilon_0^2 n^4 h^4}$$

$$KE = \frac{me^4}{8\varepsilon_0^2 n^2 h^2}$$

Total energy = KE + PE

$$= \frac{me^4}{8\varepsilon_0^2 n^2h^2} + \left(-\frac{me^4}{4\varepsilon_0^2 n^2 h^2}\right)$$

$$= -\frac{me^4}{8\varepsilon_0^2 n^2 h^2}$$

Q 5. (i) State Bohr's postulate to define stable orbits in hydrogen atom. How does de-Broglie's hypothesis explain the stability of these orbits?

(ii) A hydrogen atom initially in the ground state absorbs a photon which excites it to the $n = 4$ level. Estimate the frequency of the photon. *(CBSE 2018)*

Ans. (i) Bohr's postulate for stable orbits, states the electron in an atom, revolves around the nucleus only in those orbits for which its angular momentum is an integral multiple of $\frac{\lambda}{2\pi}$ (here, h = Planck's constant).

$$mvr = n\frac{h}{2\pi} \quad (n = 1, 2, 3, \ldots) \quad \text{....(1)}$$

As per de-Broglie hypothesis,

$$\lambda = \frac{h}{p} = \frac{h}{mv} \quad \text{....(2)}$$

For a stable orbit, we must have circumference of the orbit,

$$2\pi r = n\lambda \quad \text{(where, } n = 1, 2, 3, \ldots)$$

$$\therefore \quad 2\pi r = n\left(\frac{h}{mv}\right) \quad \text{or} \quad mvr = n\frac{h}{2\pi}$$

which is same as eq. (1).

Thus, de-Broglie hypothesis shows that formation of stationary pattern for integral 'n' gives rise to stability of the atom.

This is exactly the same as Bohr's postulate to define stable orbits.

(ii) We know that, energy of electron in nth orbit is,

$$E_n = -\frac{13.6}{n^2}\text{ eV}$$

For $n = 1$, $E_1 = -13.6$ eV

Similarly, for $n = 4$, $E_4 = -\frac{13.6}{(4)^2}\text{eV} = \frac{-13.6}{16}\text{eV}$

$\therefore$ Energy difference, $\Delta E = E_4 - E_1$

$$= \left[-\frac{13.6}{16} - (-13.6)\right] \text{ eV}$$

Also, energy of photon is

$$\Delta E = h\nu \Rightarrow \nu = \frac{\Delta E}{h}$$

Hence, $$\nu = \left(-\frac{13.6}{16} + 13.6\right) \times \frac{1.6\times10^{-19}}{6.62\times10^{-34}}$$

$\therefore$ $\nu = 3.1\times10^{15}$ Hz

TiP

Use mind map to make this type of questions easier to recall.

Q 6. The ground state energy of hydrogen atom is –13.6 eV. The photon emitted during the transition of electron from $n = 3$ to $n = 1$ state, is incident on a photosensitive material of unknown work function. The photoelectrons are emitted from the material with the maximum kinetic energy of 9 eV. Calculate the threshold wavelength of the material used.

(CBSE SQP 2022-23)

Sol. For a transition from $n = 3$ to $n = 1$ state, the energy of the emitted photon,

$$h\nu = E_2 - E_1 = 13.6\left[\frac{1}{1^2} - \frac{1}{3^2}\right]\text{eV} = 12.1 \text{ eV}$$

From Einstein's photoelectric equation, we have

$$h\nu = K_{max} + W_0$$

$\therefore$ $W_0 = h\nu - K_{max} = 12.1 - 9 = 3.1$ eV

$\therefore$ Threshold wavelength of the material used,

$$\lambda_0 = \frac{hc}{W_0} = \frac{6.62\times10^{-34}\times3\times10^{8}}{3.1\times1.6\times10^{-19}} = 4\times10^{-7} \text{ m}$$

So, the threshold wavelength of the material used $= 4 \times 10^{-7}$ m.

Q 7. The total energy of an electron in the first excited state of the hydrogen atom is about – 3.4 eV.

(i) What is the kinetic energy of the electron in this state?

(ii) What is the potential energy of the electron in this state?

(iii) Which of the answers above would change if the choice of the zero of potential energy is changed?

Sol. (i) Total energy of the electron, $E = -3.4$ eV

Kinetic energy of the electron is equal to the negative of the total energy.

$\Rightarrow$ $K = -E = -(-3.4) = +3.4$ eV

Hence, the kinetic energy of the electron in the given state is + 3.4 eV.

(ii) Potential energy (U) of the electron is equal to the negative of twice of its kinetic energy.

$\Rightarrow$ $U = -2K$

$= -2 \times 3.4 = -6.8$ eV

Hence, the potential energy of the electron in the given state is – 6.8 eV.

(iii) The potential energy of a system depends on the reference point taken. Here, the potential energy of the reference point is taken as zero. If the reference point is changed, then the value of the potential energy of the system also changes. Since, total energy is the sum of kinetic and potential energies, total energy of the system will also change.

Q 8. A photon emitted during the de-excitation of electron from a state n to the first excited state in a hydrogen atom, irradiates a metallic cathode of work function 2eV, in a photocell, with a stopping potential of 0.55 V. Obtain the value of the quantum number of the state n. *(CBSE 2019)*

Sol. Here, $\phi = 2$ eV, $\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

$$E = \frac{hc}{\lambda} = hcR\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) = \phi + KE$$

Also, $KE = eV_0$

$n_1 = 2, n_2 = n$

$$\therefore E = hcR\left(\frac{1}{4} - \frac{1}{n^2}\right) = 2\times1.6\times10^{-19} + 1.6\times10^{-19}\times0.55$$

$$\Rightarrow 6.62\times10^{-34}\times3\times10^{8}\times1.097\times10^{7}\left(\frac{1}{4} - \frac{1}{n^2}\right)$$

$$= (3.2 + 0.88)\times10^{-19}$$

$$\Rightarrow 21.786\times10^{-19}\left(\frac{1}{4} - \frac{1}{n^2}\right) = 4.08\times10^{-19}$$

$$\Rightarrow \frac{1}{4} - \frac{1}{n^2} = 0.187 \Rightarrow n = 4$$

Long Answer Type Questions

Q 1. State the postulates of Bohr's model of hydrogen atom and derive the expression for Bohr radius.

(CBSE 2020)

Ans. Postulates of Bohr model of Hydrogen atom:

Postulate-I: The electrons revolve in a circular orbit around the nucleus. The electrostatic force of attraction between the positively charged nucleus and negatively charged electrons provide necessary centripetal force for circular motion.

Postulate-II: The electrons can revolve only in certain selected orbits in which angular momentum of electrons is equal to the integral multiple $\frac{h}{2\pi}$, where h is Planck's constant. These orbits are known as stationary or permissible orbits. The electrons do not radiate energy while revolving in these orbits.

Postulate-III: When an electrons jumps from higher energy orbit to lower energy orbit, energy is radiated in the form of a quantum or photon of energy $h\nu$, which is equal to the difference of the energies of the electron in the two orbits.

Expression for Bohr radius:

From postulate-I,

Centripetal force = Electrostatic force

$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}$$

$$\therefore \quad v^2 = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{mr} \quad ...(1)$$

From postulate-II,

$$mvr = \frac{nh}{2\pi}$$

or $$v = \frac{nh}{2\pi mr}$$

or $$v^2 = \frac{n^2h^2}{4\pi^2m^2r^2} \quad ...(2)$$

Comparing eqs. (1) and (2), we get

$$\frac{1}{4\pi\varepsilon_0}\frac{e^2}{mr} = \frac{n^2h^2}{4\pi^2m^2r^2}$$

$$\therefore \quad \text{Bohr radius, } r = \frac{\varepsilon_0 n^2h^2}{\pi me^2}$$

where, m = mass of an electron,

r = radius of the circular orbit in which the electron is revolving,

v = speed of electron,

e = charge of electron

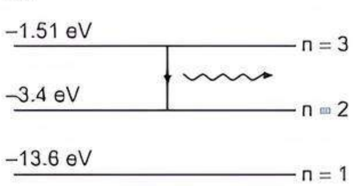
TiP

Students should carefully revise the relationship between the different variables in case of Bohr's postulates.

Q 2. (i) The ground state energy of hydrogen atom is –13.6 eV. If an electron makes a transition from an energy level –1.51 eV to –3.4 eV, calculate the wavelength of the spectral line emitted.

(ii) Using Bohr's postulates, derive the expression for the orbital period of the electron moving in the *n*th orbit of hydrogen atom. *(CBSE 2017)*

Ans. (i) Energy levels of H-atom are as shown below:



Wavelength of spectral line emitted,

$$\lambda = \frac{hc}{\Delta E}$$

$\therefore$ $hc = 1240$ eV nm,

We have, $\Delta E = E_f - E_i$

$$\Delta E = -1.51 - (-3.4) = 1.89 \text{ eV}$$

$$\lambda = \frac{1240}{1.89} = 656 \text{ nm}$$

(ii) According to Bohr's model, the centripetal force required for the revolution of electrons around the nucleus is provided by the electrostatic force of attraction between the electron and the nucleus.

If m is the mass of electron moving with a velocity v in a circular orbit of radius r, then the necessary centripetal force is

$$F = \frac{mv^2}{r} \quad ...(1)$$

Also, the electrostatic force of attraction between the nucleus of charge $(+Ze)$ and electron of charge (e^-) is

$$F = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r^2} = \frac{KZe^2}{r^2} \quad ...(2)$$

where, $K = \frac{1}{4\pi\varepsilon_0}$

From eqs. (1) and (2), we get

$$\frac{mv^2}{r} = \frac{KZe^2}{r^2}$$

$$\Rightarrow \quad r = \frac{KZe^2}{mv^2} \quad ....(3)$$

Also one of the Bohr's postulates states that the electron revolves in stationary orbits where the angular momentum of electron is an integral multiple of $\frac{h}{2\pi}$.

$$\therefore \quad mvr = \frac{nh}{2\pi}$$

Here, h is Planck's constant and n is any positive integer *i.e.*, 1, 2, 3, ...

$$\Rightarrow \quad r = \frac{nh}{2\pi vm} \quad ...(4)$$

From eqs. (3) and (4), we get

$$\frac{KZe^2}{mv^2} = \frac{nh}{2\pi mv}$$

$$\Rightarrow \quad v = \frac{2\pi KZe^2}{nh} \quad ...(5)$$

Now, we know frequency (ν) of electron in Bohr's stationary orbit is given by

$$v = r\omega = r(2\pi\nu)$$

$$\nu = \frac{v}{2\pi r}$$

From eq. (5), we get

$$\nu = \frac{v}{2\pi r} = \frac{2\pi KZe^2}{nh \cdot 2\pi r} = \frac{KZe^2}{nhr}$$

For hydrogen atom $Z = 1$, therefore

$$\nu = \frac{Ke^2}{nhr}$$

Above expression is the orbital frequency of electron moving in the nth orbit of hydrogen atom.

Now, we know the orbital period (T) of the electron moving in the nth orbit of hydrogen atom is $T = \frac{1}{\nu} = \frac{nhr}{Ke^2}$.

Q 3. A 12.75 eV beam is used to bombard gaseous hydrogen at room temperature. What series of wavelengths will be emitted? *(NCERT EXERCISE)*

Ans. Energy of the hydrogen atom in lowest energy-level $E = -13.6$ eV. Now it is bombarded by a beam of electrons of energy = 12.75 eV.

The energy absorbed by it is $E_n = -13.6 \text{ eV} + 12.75 \text{ eV}$
$= -0.85$ eV

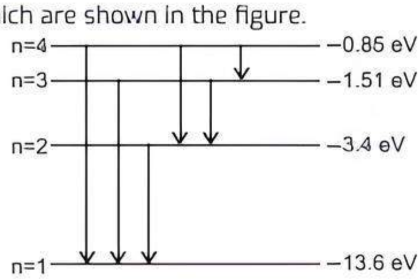
Now, $E_n = \frac{-13.6 \text{ eV}}{n^2} = -0.85 \text{ eV}$

or $n^2 = \frac{13.6}{0.85} = 16$

or $n = 4$

Possible transitions $= \frac{n(n-1)}{2} = \frac{4 \times (4-1)}{2} = 6$

which are shown in the figure.



The wavelengths emitted are

$$\frac{hc}{\lambda} = \Delta E$$

or $\lambda = \frac{hc}{\Delta E} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{\Delta E \text{ (in Joule)}}$

$$= \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{1.6 \times 10^{-19} \times (\Delta E \text{ in eV})} \text{ m} = \frac{12.43}{\Delta E \text{ in eV}} \times 10^{-7} \text{ m}$$

For n_4 to n_1, $\Delta E = [-0.85 - (-13.6)]$ eV
$= 12.75$ eV,

$$\lambda_1 = \frac{12.43}{12.75} \times 10^{-7} \text{ m} = 974.9 \text{ Å}$$

n_3 to n_1, $\Delta E = [-1.51 - (-13.6)]$ eV $= 12.19$ eV,

$$\lambda_2 = \frac{12.43}{12.19} \times 10^{-7} \text{ m} = 1019.6 \text{ Å}$$

n_2 to n_1, $\Delta E = [-3.4 - (-13.6)]$ eV $= 10.2$ eV,

$$\lambda_3 = \frac{12.43}{10.2} \times 10^{-7} \text{ m} = 1218.6 \text{ Å}$$

n_4 to n_2, $\Delta E = [-0.85 - (-3.4)]$ eV $= 2.55$ eV,

$$\lambda_4 = \frac{12.43}{2.55} \times 10^{-7} \text{ m} = 4874.5 \text{ Å}$$

n_3 to n_2, $\Delta E = [-1.51 - (-3.4)]$ eV $= 1.89$ eV,

$$\lambda_5 = \frac{12.43}{1.89} \times 10^{-7} \text{ m} = 6576.7 \text{ Å}$$

n_4 to n_3, $\Delta E = [-0.85 - (-1.51)]$ eV $= 0.66$ eV,

$$\lambda_6 = \frac{12.43}{0.66} \times 10^{-7} \text{ m} = 18833 \text{ Å}$$

The wavelengths emitted will be 974.9 Å, 1019.6 Å, 1218.6 Å, 4874.5 Å, 6576.7 Å, 18833 Å respectively.

Chapter Test

Multiple Choice Questions

Q 1. In Bohr's model of an atom, which of the following is an integral multiple of $\frac{h}{2\pi}$?

a. Kinetic energy
b. Radius of an atom
c. Potential energy
d. Angular momentum

Q 2. The angular momentum of the electron in the nth allowed orbit is:

a. $\frac{3h}{2\pi}$
b. $\frac{h}{2\pi}$
c. $\frac{2h}{\pi}$
d. $\frac{nh}{2\pi}$

Assertion and Reason Type Questions

Directions (Q.Nos. 3-4): *In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as:*

a. Both Assertion (A) and Reason (R) are true and Reason (R)is the correct explanation of Assertion (A).
b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).
c. Assertion (A) is true but Reason (R) is false.
d. Both Assertion (A) and Reason (R) are false.

Q 3. Assertion (A): Bohr's third postulate states that the stationary orbits are those for which the angular momentum is some integral multiple of $\frac{h}{2\pi}$.

Reason (R): Linear momentum of the electron in the atom is quantised.

Q 4. Assertion (A): The whole mass of the atom is considered in the nucleus.

Reason (R): The mass of a nucleus can be either less than or more than the sum of the masses of nucleons present in it.

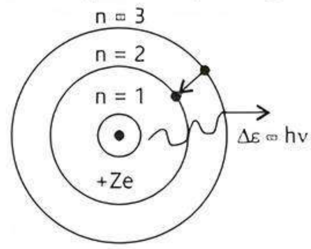
Fill in the blanks

Q 5. The Bohr's model is applicable to .... atom.

Q 6. The scattering angle will decreases with the .... in impact parameter.

Case Study Based Question

Q 7. Niels Bohr introduced the atomic Hydrogen model in 1913. He described it as a positively charged nucleus, comprised of protons and neutrons, surrounded by a negatively charged electron cloud. In the model, electrons orbit the nucleus in atomic shells. The atom is held together by electrostatic forces between the positive nucleus and negative surroundings.



Bohr correctly proposed that the energy and radii of the orbits of electrons in atoms are quantised, with energy for transitions between orbits given by $DE = h\nu = E_i - E_f$, where DE is the change in energy between the initial and final orbits and $h\nu$ is the energy of an absorbed or emitted photon.

Read the given passage carefully and give the answer of the following questions:

(i) What is the angular speed of the electron in the nth orbit of Bohr's hydrogen atom?

(ii) When electron jumps from $n = 4$ level to $n = 1$ level, then what will be the change in the angular momentum of electron?

(iii) Find the energy in lowest Bohr orbit of hydrogen atom.

(iv) Which postulate of the Bohr model led to the quantisation of energy of the hydrogen atom?

Very Short Answer Type Questions

Q 8. In accordance with the Bohr's model, what will be the quantum number that characteristics the earth's revolution around the sun in an orbit of radius 1.5×10^{11} m with orbital speed 3×10^4 ms^{-1}?

(Take mass of earth = 6×10^{24} kg)

Q 9. The radius of innermost orbit of a hydrogen atom is 5.3×10^{-11} m. What are the radii of $n = 2$ orbit?

Short Answer Type-I Questions

Q 10. It is found experimentally that 13.6 eV energy is required to separate a hydrogen atom into a proton and an electron. Compute the orbital radius and the velocity of the electron in a hydrogen atom.

Q 11. What is meant by ionisation energy? Write its value for hydrogen atom. *(CBSE 2023)*

Short Answer Type-II Questions

Q 12. Using Rutherford model of the atom, derive the expression for the total energy of the electron in hydrogen atom. What is the significance of total negative energy possessed by the electron?

Q 13. Draw a schematic arrangement of the Geiger-Marsden experiment. How did the scattering of α-particles of a thin foil of gold provide an important way to determine an upper limit on the size of the nucleus? Explain briefly.

Q 14. An α-particle after passing through a potential difference of 2×10^6 V falls on a silver foil. The atomic number of silver is 47. Calculate:

(i) the kinetic energy of the α-particle at the time of falling on the foil.

(ii) the kinetic energy of the α-particle at a distance of 5×10^{-14} m from the silver nucleus.

(iii) the shortest distance from the nucleus of silver to which the α-particles reaches.

($e = 1.6 \times 10^{-19}$ C and $\frac{1}{4\pi\varepsilon_0} = 9.0\times10^9$ Nm^{2}C^{-2})

Long Answer Type Questions

Q 15. Answer the following questions, which help you to understand the difference between Thomson's model and Rutherford's model better:

(i) Is the average angle of deflection of α-particles by a thin gold-foil predicted by Thomson's model much less, about the same or much greater than that predicted by Rutherford's model?

(ii) Is the probability of backward scattering (*i.e.*, scattering of α-particles at angles greater than 90°) predicted by Thomson's model much less, about the same or much greater than that predicted by Rutherford's model?

(iii) Keeping other factors fixed, it is found experimentally that for small thickness t, the number of α-particles scattered at moderate angles is proportional to t. What clue does this linear dependence on t provide?

(iv) In which model is it completely wrong to ignore multiple scattering for the calculation of average angle of scattering of α-particles by a thin foil?

Q 16. In a Geiger-Marsden experiment, what is the distance of closest approach to the nucleus of a 7.7 MeV α-particle before it comes momentarily to rest and reverses its direction?